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N-photon factorial moments for superposition of coherent and chaotic fields

Abstract. The first and the second N-photon factorial moments for the superposition of coherent and chaotic fields and the third N-photon factorial moment for the chaotic field are given for arbitrary spectrum and counting time intervals.

There has been recent interest in multi-photon counting statistics (Teich and Diament 1969, Jaiswal and Agarwal 1969, Barashev 1970a,b, Mišta and Peřina 1970). It was shown that in a counting experiment with an N-photon detector and quasi-monochromatic stationary field one can measure the N-photon counting distribution $p_N(n, T)$ and its factorial moments which are equal to

$$\sum_{n=0}^{\infty} p_N(n, T) \frac{n!}{(n-k)!} = \langle W_N^k \rangle = \beta_N^k \int_0^T \!\!\dots\!\! \int \langle I^N(t_1) I^N(t_2) \dots I^N(t_k) \rangle \, \mathrm{d}t_1 \dots \mathrm{d}t_k \qquad (1)$$

where $I(t)$ is an instantaneous intensity of incident field, β_N is the photoefficiency of the N-photon detector and T is the counting time interval.

If the incident field is the linearly polarized superposition of coherent and chaotic fields then it is possible to calculate moments (1) by means of the slightly modified graphical method proposed in Peřina and Mišta (1968). For $k = 1$ and $k = 2$ we obtain

$$\langle W_N \rangle = \beta_N T \sum_{l=0}^{N} \binom{N}{l}^2 l! \, I_{\mathrm{c}}{}^{N-l} I_{\mathrm{Ch}}{}^{l} \qquad (2)$$

and

$$\langle W_N{}^2 \rangle = \beta_N{}^2 (N!)^4 \sum_{l_1=0}^{N} \sum_{l_2=0}^{N} \sum_{l_3=0}^{\min\{(N-l_1),(N-l_2)\}} \sum_{l_4=0} I_{\mathrm{Ch}}^{l_1+l_2+l_3+l_4} I_{\mathrm{c}}^{2N-l_1-l_2-l_3-l_4}$$

$$\left\{ \prod_{i=1}^{4} l_i! \, (N-l_1-l_3)! \, (N-l_1-l_4)! \, (N-l_2-l_3)! \, (N-l_2-l_4)! \right\}^{-1}$$

$$\int_0^{\mathrm{T}} \!\! \int \gamma_{\mathrm{Ch}}{}^{l_3}(t_1-t_2) \gamma_{\mathrm{Ch}}{}^{l_4}(t_2-t_1) \gamma_{\mathrm{c}}{}^{l_3}(t_2-t_1) \gamma_{\mathrm{c}}{}^{l_4}(t_1-t_2) \, \mathrm{d}t_1 \, \mathrm{d}t_2 \qquad (3)$$

where $\bar{I}_{\mathrm{Ch}}$ and $\bar{I}_{\mathrm{c}}$ are the mean intensities of chaotic and coherent fields, γ_{Ch} and $\gamma_{\mathrm{c}}(\tau) = \mathrm{e}^{\mathrm{i}\omega_{\mathrm{c}}\tau}$ are normalized correlation functions. If the chaotic field has Lorentzian spectral profile then $\gamma_{\mathrm{Ch}}(\tau) = \exp(-\Gamma|\tau| + \mathrm{i}\omega_0\tau)$ and the integral on the right hand side of equation (3) is equal to $T^2F\{(l_3+l_4)\gamma, |l_3-l_4|\Omega\}$, where

$$F(\gamma, \Omega) = \frac{2\gamma}{\gamma^2+\Omega^2} + \frac{2(\Omega^2-\gamma^2)}{(\Omega^2+\gamma^2)^2} + \frac{2\mathrm{e}^{-\gamma}\{(\gamma^2-\Omega^2)\cos\Omega - 2\gamma\Omega\sin\Omega\}}{(\Omega^2+\gamma^2)^2} \tag{4}$$

and $\gamma = \Gamma T$, $\Omega = (\omega_0 - \omega_{\mathrm{c}})T$. This result for $N = 2$ has been given in Mista and Perina (1970).

When only chaotic field is present then $\bar{I}_{\mathrm{c}} = 0$ and from (3) one immediately obtains

$$\langle W_N{}^2 \rangle = \beta_N{}^2(N!)^2\bar{I}_{\mathrm{Ch}}{}^{2N} \sum_{l=0}^{N} \binom{N}{l}^2 \int_0^T\!\!\int |\gamma_{\mathrm{Ch}}(t_1-t_2)|^{2l}\,\mathrm{d}t_1\,\mathrm{d}t_2. \tag{5}$$

If $\tau_{\mathrm{c}} \gg T$, where τ_{c} is coherence time of incident light, then we have from (5)

$$\langle W_N{}^2 \rangle = \beta_N{}^2(N!)^2\bar{I}_{\mathrm{Ch}}{}^{2N} \sum_{l=0}^{N} \binom{N}{l}^2 T^2 = \beta_N{}^2T^2\bar{I}_{\mathrm{Ch}}{}^{2N}(2N)!. \tag{6}$$

From (6) and (2) where $\bar{I}_{\mathrm{c}} = 0$ is put it follows that $\langle W_N{}^2\rangle/\langle W_N\rangle^2 = (2N)!/(N!)^2$ (Barashev 1970b).

The expression for the kth moment ($k > 2$) corresponding to (3) should also be obtained but it is of rather complicated structure (it contains k^2 summing indices). Hence we report only the third moment for the chaotic field which is equal to

$$\begin{aligned}\langle W_N{}^3 \rangle = {} & \beta_N{}^3(N!)^6\bar{I}_{\mathrm{Ch}}{}^{3N} \sum_{l_1=0}^{N} \sum_{l_2=0}^{N} \sum_{l_3,l_4\geqslant 0} \left\{ \prod_{i=1}^{4} l_i! \left(-N + \sum_{i=1}^{4} l_i\right)!\,(N-l_1-l_4)! \right. \\ & \left. \times (N-l_2-l_4)!\,(N-l_1-l_3)!\,(N-l_2-l_3)! \right\}^{-1} \\ & \times \int_0^T\!\!\int\!\!\int \gamma_{\mathrm{Ch}}{}^{l_4}(t_1-t_2)\gamma_{\mathrm{Ch}}{}^{l_3}(t_2-t_1)\gamma_{\mathrm{Ch}}{}^{N-l_1-l_4}(t_1-t_3) \\ & \times \gamma_{\mathrm{Ch}}^{N-l_1-l_3}(t_3-t_1)\gamma_{\mathrm{Ch}}^{N-l_2-l_3}(t_2-t_3)\gamma_{\mathrm{Ch}}^{N-l_2-l_4}(t_3-t_2)\,\mathrm{d}t_1\,\mathrm{d}t_2\,\mathrm{d}t_3\end{aligned} \tag{7}$$

although this moment for the superposition has been written down, too. (The sums in (7) over l_3, l_4 must be taken in such a way that the factorials are well defined there similarly as in (3).)

Equation (7) for $N = 2$ has been used in Mišta and Peřina (1970) for calculation of the third moment of the Lorentzian–Gaussian field.

Department of Theoretical Physics and Astronomy
Palacký University
Olomouc, Czechoslovakia

L Mišta
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